INVITED PAPER

The Fourth Element

This tutorial clarifies why the memristor is the fourth circuit element, and why its fingerprint is a continuum of pinched hysteresis loops at all frequencies, and for all initial conditions.

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ABSTRACT | This tutorial clarifies the axiomatic definition of $(v^{(\alpha)}, i^{(\beta)})$ circuit elements via a lookup table dubbed an *A-pad*, of admissible (v, i) signals measured via Gedanken probing circuits. The $(v^{(\alpha)}, i^{(\beta)})$ elements are ordered via a complexity metric. Under this metric, the memristor emerges naturally as the fourth element, characterized by a state-dependent Ohm's law. A logical generalization to memristive devices reveals a common fingerprint consisting of a dense continuum of pinched hysteresis loops whose area decreases with the frequency ω and tends to a straight line as $\omega \to \infty$, for all bipolar periodic signals and for all initial conditions. This common fingerprint suggests that the term memristor be used henceforth as a moniker for memristive devices.

KEYWORDS | Fourth element; Hodgkin–Huxley axon circuit model; memristor; pinched hystersis loops; $\alpha - \beta$ circuit elements

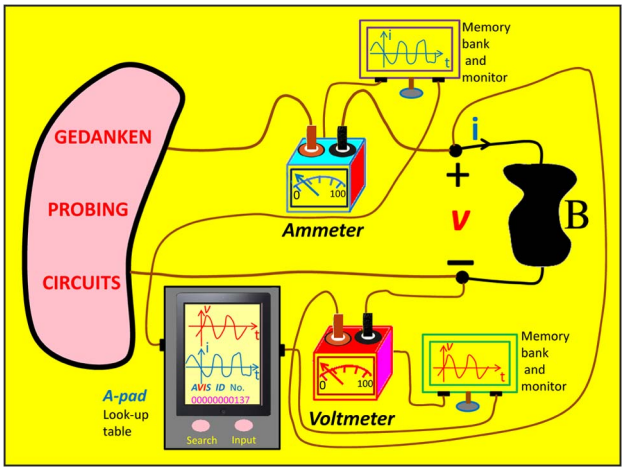

Fig. 1. ***Axiomatic definition of a two-terminal circuit element.***

I. AXIOMATIC DEFINITION OF CIRCUITS ELEMENTS

How do you characterize a two-terminal "black box" B such that its response to any electrical signal can be predicted? Since you are not allowed to peek inside B your only recourse is to carry out measurements by probing B with all possible electrical circuits, containing arbitrary interconnections of circuit elements, such as resistors, capacitors, inductors, diodes, transistors, op amps, batteries, voltage and current sources with arbitrary time functions, etc. We will henceforth call such circuits "Gedanken probing circuits," as depicted in the Gedanken experimental setup shown in Fig. 1. Let us insert an instrument called an ammeter in series with the top wire to record a time function $i(t)$ called the current in amperes entering the top terminal [labeled by a plus $(+)$ sign]. Next let us connect an instrument called a voltmeter across B to record a time function $v(t)$ called the voltage in volts across the plus–minus terminals of B.[1] Let us call $(v(t), i(t))$, $-\infty < t < \infty$, an admissible (v, i) signal of B. The recorded list

$$B(v,i) \triangleq \{(v_1(t), i_1(t)), (v_2(t), i_2(t)), \ldots, (v_n(t), i_n(t)), \ldots\} \quad (1)$$

of admissible (v, i) signals (AVIS) from all possible Gedanken probing circuits constitutes the complete characterization of the two-terminal black box B in the sense that given any voltage signal $\tilde{v}(t)$, or current signal $\tilde{i}(t)$, one

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[1] Observe that the voltage v and the current i are defined axiomatically via two instruments called voltmeter and ammeter, without invoking any physical concepts such as electric field, magnetic field, charge, flux linkages, etc. One does not even have to know how a voltmeter, or an ammeter, works. They are just names assigned to the instruments.

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can search the AVIS "memory bank," henceforth called the *AVIS-pad* of B or just *A-pad*, and identify the unique admissible signals $(\tilde{v}(t), \tilde{i}(t))$ being sought. The *A-pad* must contain this entry in its memory bank because the signal is associated with some circuit connected to B, and this circuit is a Gedanken probing circuit, by definition. The *A-pad* is just a lookup table containing all admissible (v, i) signals of B. Observe that the *A-pad* is in general an infinitely long pad containing infinitely many pairs of admissible signal waveforms $(v(t), i(t))$ of B, as depicted in Fig. 1.

The above Gedanken experiment is only a thought experiment. However, for a large number of real-world two-terminal devices, the *A-pad* for B can be generated via equations.

Example 1 (Ohm's Law): A very small subset of all two-terminal black boxes is characterized by an *A-pad* that satisfies Ohm's Law; namely

$$v = Ri \quad \text{or} \quad i = Gv \tag{2}$$

where R is called the resistance in ohms (Ω) of B and G is called the conductance in siemens (S) of B. In this case

$$\text{AVIS} = \{(Ri_1(t), i_1(t)), (Ri_2(t), i_2(t)), \ldots, (Ri_n(t), i_n(t)), \ldots\} \tag{3}$$

can be reconstructed by (2). When Ohm's law is written with i as the independent variable, namely, $v = Ri$, it is called current controlled. If it is written in the form $i = Gv$, it is called voltage controlled. Often it is more convenient to recast (2) in the implicit form

$$f_R(v, i) = v - Ri = 0. \tag{4}$$

Since (4) is neither a function of v nor of i, it is called a relation in mathematics. In nonlinear circuit theory, it is called a constitutive relation [2]–[4]. Observe that the constitutive relation is just a compact formula, or algorithm, for generating the *A-pad* of B.

Example 2: Suppose the *A-pad* of the two-terminal black box B in Fig. 1 can be written in the form

$$AVIS = \left\{\left(v_1, v_1 + \frac{1}{3}v_1^3\right), \left(v_2, v_2 + \frac{1}{3}v_2^3\right), \ldots, \left(v_n, v_n + \frac{1}{3}v_n^3\right), \ldots\right\} \tag{5}$$

for all possible voltage signals $v(t) = v_1(t), v(t) = v_2(t), \ldots, v(t) = v_n(t), \ldots$, then the *A-pad* of B can be generated by the much more compact constitutive relation

$$f_R(v, i) = v + \frac{1}{3}v^3 - i = 0. \tag{6}$$

Since both (4) of Example 1 and (6) of Example 2 involve the same pair of circuit variables (voltage, current), and since all two-terminal devices that can be characterized by a constitutive relation

$$f_R(v, i) = 0 \tag{7}$$

between the variable pair (v, i) can be proved to be dissipative (or passive) if $v \times i > 0$ for all (v, i) listed in the *A-pad*, this class of two-terminal elements is called resistors [2]–[4].

Example 3: Most two-terminal black boxes cannot be described by a constitutive relation between the variable pair (v, i). However, another important subclass can be expressed by a relationship between the variable pair (v, q), where

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau = q_0 + \int_{t_0}^{t} i(\tau)d\tau \tag{8}$$

and

$$q_0 \triangleq \int_{-\infty}^{t_0} i(\tau)d\tau \tag{9}$$

is called the initial state[2] of $q(t)$ at the initial time $t = t_0$. This subclass of two-terminal black boxes can be characterized by a collection of admissible signals between the variable pair (v, q), namely

$$B(v, q) = \{(v_1(t), q_1(t)), (v_2(t), q_2(t)), \ldots, (v_n(t), q_n(t)), \ldots\} \tag{10}$$

[2]In practice one can never know the precise signal $i(t)$ over the infinite past. Rather we can only set up our measurements to begin at some initial time $t = t_0$. Consequently, the initial condition q_0 in (8) represents a summary of the past memory of $q(t)$ measured at $t = t_0$.

where

$$q = Cv \tag{11}$$

and C is a constant called the capacitance of B. Equation (11) is the constitutive relation of B because we can generate the corresponding AVIS $(v(t), i(t))$ via (8), namely

$$i(t) = \frac{dq(t)}{dt}. \tag{12}$$

Indeed, any relationship

$$q = f_C(v) \tag{13}$$

is a valid constitutive relation and this class of two-terminal devices is called capacitors.

By the same reasoning, the constitutive relation

$$\varphi = f_L(i) \tag{14}$$

involving the variable pair (i, φ) defines a third subclass of two-terminal devices called inductors, where

$$\varphi(t) = \int_{-\infty}^{t} v(\tau)d\tau = \varphi_0 + \int_{t_0}^{t} v(\tau)d\tau. \tag{15}$$

Observe that the above three classes of basic circuit elements, called resistors, capacitors, and inductors, are defined axiomatically, via a constitutive relation between a pair of variables chosen from $\{v, i, q, \varphi\}$. There are six different pairs that can be formed from these four variables, namely

$$\{(v, \varphi), (i, q), (v, i), (v, q), (i, \varphi), (\varphi, q)\}. \tag{16}$$

The first two pairs (v, φ) and (i, q) are already related via (15) and (8), respectively, and are not constitutive relations because they cannot predict the corresponding current $i(t)$ and voltage $v(t)$. However, the last pair (φ, q) defines yet another constitutive relation since given any admissible signals $(\varphi(t), q(t))$, one can recover the corresponding $(v(t), i(t))$ via (15) and (8). For logical consistency, and symmetry considerations, it is necessary to define a fourth circuit element [1] via the constitutive relation

$$f_M(\varphi, q) = 0 \tag{17}$$

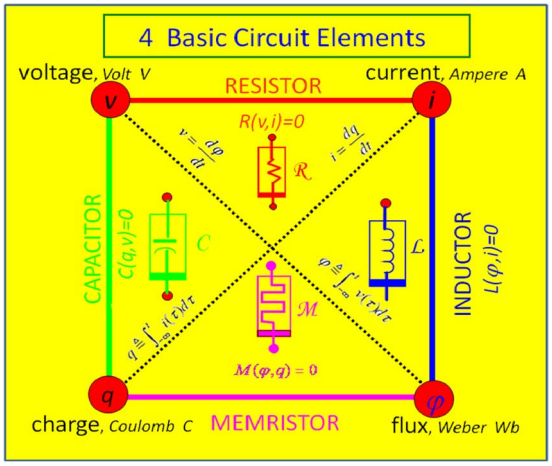

Fig. 2. *Four axiomatically defined circuit elements.*

between the variables φ and q. This element was postulated and named the memristor (acronym for *mem*ory re*sistor*) in [5]. A physical approximation of such an element has been fabricated in 2008 as a TiO_2 nano device by Williams' group at HP [6]. The above axiomatic definitions of the four basic circuit elements are summarized in Fig. 2, along with their respective symbols [7]. Note that the standard symbols for resistor, capacitor, and inductor are enclosed by a thin rectangle with a dark band at the bottom because it is essential to distinguish the reference polarity of each nonlinear element if its constitutive relation is *not* odd symmetric.

We wish to stress that although the symbols of q and φ in Fig. 2 are given the names charge and flux, respectively, they need not be associated with a real physical charge as in the case of a classical capacitor built by sandwiching a pair of parallel plates between an insulator, or a real physical flux as in the case of a classical inductor built by winding a copper wire around an iron core.

II. $(v^{(\alpha)} - i^{(\beta)})$ CIRCUIT ELEMENTS

Let us introduce the notations [4]

$$v^{(\alpha)}(t) \triangleq \begin{cases} \frac{d^\alpha v(t)}{dt^\alpha}, & \text{if} \quad \alpha = 1, 2, \ldots \infty \\ v(t), & \text{if} \quad \alpha = 0 \\ \int_{-\infty}^{t} v(\tau)\, d\tau, & \text{if} \quad \alpha = -1 \\ \int_{-\infty}^{t} \int_{-\infty}^{\tau_{|\alpha|}} \cdots & \\ \int_{-\infty}^{\tau_2} v(\tau_1) d\tau_1 d\tau_2 \cdots d\tau_{|\alpha|}, & \text{if } \alpha = -2, -3, \ldots \infty \end{cases} \tag{18}$$

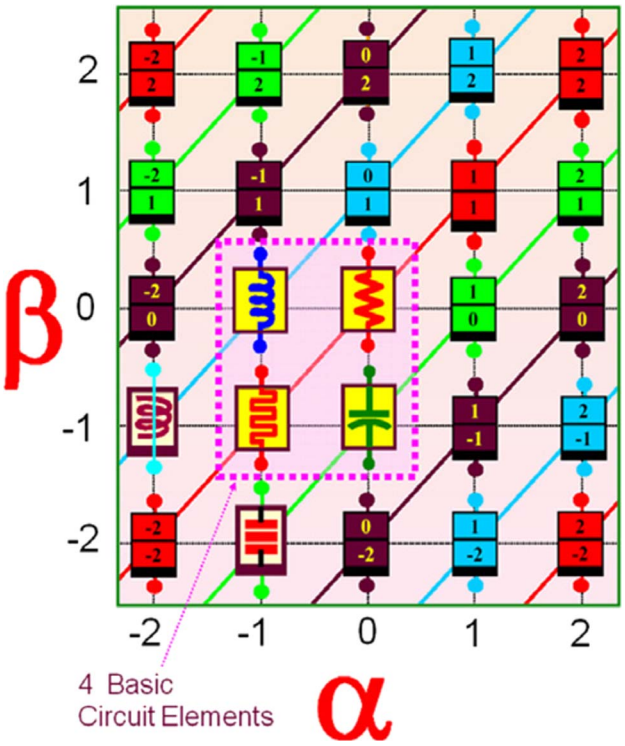

Fig. 3. *The first 25* (α, β) *circuit elements,* $-2 \le \alpha \le 2, -2 \le \beta \le 2$.

and

$$i^{(\beta)}(t) \triangleq \begin{cases} \frac{d^{\beta} i(t)}{dt^{\beta}}, & \text{if } \beta = 1, 2, \ldots \infty \\ i(t), & \text{if } \beta = 0 \\ \int_{-\infty}^{t} i(\tau) d\tau, & \text{if } \beta = -1 \\ \int_{-\infty}^{t} \int_{-\infty}^{\tau_{|\beta|}} \cdots & \\ \int_{-\infty}^{\tau_2} i(\tau_1) d\tau_1 d\tau_2 \cdots d\tau_{|\beta|}, & \text{if } \beta = -2, -3, \ldots \infty \end{cases} \tag{19}$$

where $|\alpha|$ and $|\beta|$ are integers. Let us identify a $(v^{(0)}, i^{(0)})$ element as a resistor, a $(v^{(0)}, i^{(-1)})$ element as a capacitor, a $(v^{(-1)}, i^{(0)})$ element as an inductor, and a $(v^{(-1)}, i^{(-1)})$ element as a memristor. Using this notation, we can define an infinite family of circuit elements, each one identified by its element code $(v^{(\alpha)} - i^{(\beta)})$, and referred to simply as an (α, β) element.

The first 25 (α, β) elements are listed in Fig. 3, each coded by an integer pair (α, β), and identified by a rectangular box where "α" and "β" are printed on the "top" and at the "bottom," respectively. Each (α, β) element is located at the intersection between a vertical line through α and a horizontal line through β. The four circuit element symbols shown in Fig. 2 are printed in their corresponding locations in Fig. 3. The two elements $(\alpha, \beta) = (-1, -2)$ and $(\alpha, \beta) = (-2, -1)$ are called memcapacitor and meminductor, respectively [11], and are identified by their corresponding symbols.

The above infinite family of circuit elements is defined not for the sake of generality. Rather, these circuit elements are essential for developing a rigorous mathematical theory of nonlinear circuits in the sense that if one excludes all elements with $|\alpha| > k$ and $|\beta| > k$, for any finite integer k, then one can construct hypothetical circuits whose solutions do not exist after certain finite times $t \ge T_k$ due to the presence of a "singularity" called an impasse point [2], [3], [10]. It is unlikely, however, that (α, β) elements with $|\alpha| > 2$ and $|\beta| > 2$ will be needed in modeling most real-world devices.

It can be proved that any (α, β) element with $|\alpha| + |\beta| > 2$ is active in the sense that it can be built only with active components, such as transistors and op amps, which requires a power supply. Finally, we remark that every (α, β) element can be built by the same procedure illustrated in [2], [5], and [12] using a family of linear active two ports called mutators. They can also be emulated via various off-the-shelf digital components [13], or by programmable software interfaced with analog-to-digital (A/D) and digital-to-analog (D/A) converters.

III. COMPLEXITY METRIC OF CIRCUIT ELEMENTS

For each (α, β) element, let

$$\chi \triangleq |\alpha| + |\beta| \tag{20}$$

be its associated complexity metric [9]. For example, $\chi(0, 0) = 0$ for a resistor, $\chi(0, -1) = 1$ for a capacitor, $\chi(-1, 0) = 1$ for an inductor, $\chi(-1, -1) = 2$ for a memristor, $\chi(-1, -2) = 3$ for a memcapacitor, and $\chi(-2, -1) = 3$ for a meminductor. If one associates the vertical and horizontal lines passing through the (α, β) elements in Fig. 3 as streets of Manhattan, New York City, then the complexity metric $\chi(\alpha, \beta)$ of an (α, β) element gives a measure of its distance from the resistor $(\alpha, \beta) = (0, 0)$. The larger the metric $\chi(\alpha, \beta)$, the farther it is from the resistor.

The complexity metric measures not just the distance of (α, β) element from the resistor, but also the minimum number of capacitors (or inductors) needed to build an (α, β) element using off-the-shelf components. For example, a minimum of one capacitor along with active elements such as transistors and op amps are needed to build a memristor while a minimum of two capacitors is needed to build a meminductor. From a mathematical perspective, the larger the complexity metric, the higher is the dimension of the *state space* and the larger is the number of nonlinear differential equations and exotic dynamical phenomena that can emerge.

Based on any of the above measures of complexity, the four elements depicted in Fig. 3 are indeed the simplest circuit elements, with the memristor ranked as the fourth element in increasing complexity.

IV. FINGERPRINT OF MEMRISTORS

The formal mathematical definition of the memristor is given in [5], along with its circuit-theoretic properties. Here we recall that the memristor is defined by a collection of all admissible $(\varphi(t), q(t))$ signals, namely, an *A-pad* listing all $(v(t), i(t))$ signals measured from all admissible "Gedanken probing circuits" (Fig. 1) and which can be completely reproduced by the constitutive relation (17).

For example, a charge-controlled memristor can be defined by

$$\varphi = f_M(q) \tag{21}$$

where $f_M(.)$ is a piecewise-differentiable function [9]. In this case, we can generate all $(v(t), i(t))$ from the *A-pad* via the following q-dependent Ohm's law:

$$\boxed{\begin{aligned} v &= R(q)i \qquad (22a)\\ R(q) &\triangleq \frac{df_M(q)}{dq}. \qquad (22b)\end{aligned}}$$

The function $R(q)$ is called the memristance (acronym for memory resistance) where

$$R(q) \geq 0 \tag{23}$$

for all passive memristors [2].

Now observe from (8) that since

$$\frac{dq}{dt} = 0, \qquad \text{when } i = 0 \tag{24}$$

the memristor can assume a continuous range of distinct equilibrium states

$$q = q(t_0), \qquad t \geq t_0 \tag{25}$$

when the power is switched off at any time $t = t_0$. It follows that the memristor can be used as a nonvolatile analog memory. In particular, it can be used as a nonvolatile binary memory where two sufficiently different values of resistance are chosen to code the binary states "0" and "1," respectively. Because the HP memristor reported in [6] as well as in many other nanodevices [14] can be scaled down to atomic dimensions, the memristor offers immense potentials for an ultralow-power and ultradense nonvolatile memory technology that could replace flash memories and dynamic random-access memories (DRAMs).

An incisive analysis of (22) reveals that the nonvolatile memory property possessed by the memristor is a direct consequence of its state-dependent Ohm's law. Moreover, all circuit–theoretic properties possessed by the memristor are preserved if we generalize (22) to the form [8]

$$\boxed{\begin{aligned} v &= R(\boldsymbol{x}, i)i \qquad (26a)\\ \frac{d\boldsymbol{x}}{dt} &= \mathbf{f}(\boldsymbol{x}, i). \qquad (26b)\end{aligned}}$$

The generalized memristor defined in (26) is dubbed a memristive device in [8] where $x = (x_1, x_2, \ldots, x_n)$ denotes n states variables $x_1, x_2, \ldots, x_n$, which do not depend on any external voltages or currents. However, since both (22) and (26) are endowed with the same *circuit–theoretic* properties, it is more convenient and logical to refer to both equations as defining a memristor. In rare events where a distinction may be desirable, one can refer to (22) as defining an "ideal memristor."

The most important common property of (22) and (26) is that the loci (i.e., Lissajous figure) of $(v(t), i(t))$ due to any periodic current source $i(t)$, or periodic voltage source $v(t)$, which assumes both positive and negative values, must always be pinched at the origin in the sense that $(v, i) = (0, 0)$ must always lie on the (v, i)-loci, called a pinched hysteresis loop in the literature [14]. We wish to stress that (22) and (26) imply that the pinched hysteresis loop phenomenon of the memristor must hold for any periodic signal $v(t)$, or $i(t)$, that assumes both positive and negative values, as well as for any initial condition used to integrate the differential equations to obtain the corresponding steady state $i(t)$ and $v(t)$, respectively.

Another unique property shared by all memristor hysteresis loops is that for every given periodic function $i = f(t)$ [where $f(\bullet)$ assumes both positive and negative values], and for any initial state $\mathbf{x}(0)$, the area enclosed within the part of the pinched hysteresis loop in the first quadrant, and the third quadrant, of the $v - i$ plane shrinks continuously as the frequency ω increases, and the hysteresis loop tends to a straight line through the origin as ω tends to ∞.

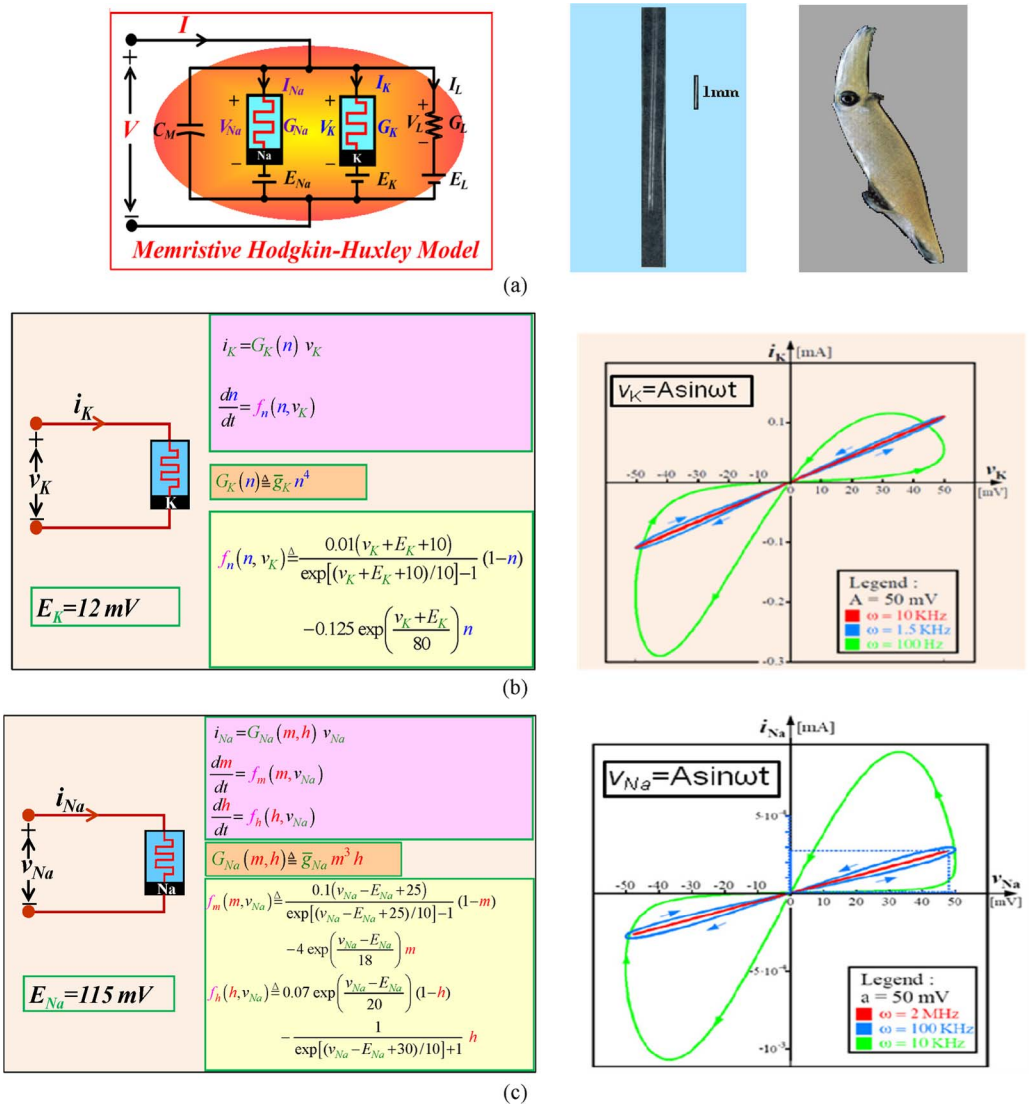

Fig. 4. ***Hodgkin-Huxley Axon. (a) Memristive Hodgkin–Huxley circuit model of giant axon (center) of North Atlantic squid Loligo (right). (b) Postassium ion-channel memristor and its pinched hysteresis loops. (c) Sodium ion-channel memristor and its pinched hysteresis loops [39].***

The above dense continuum of pinched hysteresis loops, as well as their slope-limiting phenomenon as $\omega \rightarrow \infty$, must hold for all memristors. Any purported system which may exhibit a pinched hysteresis loop but which violates the above continuum and frequency-dependent slope-limiting memristor fingerprint is not a memristor. The reader is referred to [15] for several contrived examples which fail the above "memristor fingerprint test."

We end this tutorial by pointing out that not all memristors are nonvolatile memories. In fact, there is an even larger class of locally active memristors [2], [4], [10], which exhibit many exotic nonlinear dynamical phenomena. A very interesting and scientifically significant example is the classic Hodgkin–Huxley axon circuit model of the squid giant axon.[3] Notwithstanding the immense importance of their circuit model, Hodgkin and Huxley had erroneously named two circuit elements in their model associated with the potassium ion, and the sodium ion, respectively, as *time-varying* conductances. This mistaken identity has led to numerous confusions and paradoxes ever since the publications of their classic axon circuit model [16]. Well-known physiologists were puzzled by experimentally observed rectification phenomenon as well as gigantic inductances that could not exist

[3]Hodgkin and Huxley were awarded the 1965 Nobel Prize in physiology for their derivation of the circuit shown in Fig. 4(a), where the two memristors were drawn as time-varying resistors in [16, Fig. 1, p. 501].

within the soft tissues of the brain. The following quotation from Cole, an eminent physiologist and recipient of the 1967 U.S. National Medal of Science, is a case in point:

> The suggestion of an inductive reactance anywhere in the system was shocking to point of being unbelievable [18, p. 78].

We have solved the above conundrum, and many other hitherto unresolved paradoxes associated with the Hodgkin–Huxley axon, by showing the Hodgkin–Huxley *time-varying* potassium conductance is in fact a first-order memristor, and the Hodgkin–Huxley *time-varying* sodium conductance is in fact a second-order memristor, as defined in Fig. 4(b) and (c), respectively [39]. Also depicted in Fig. 4 are the pinched hysteresis loops associated with each memristor. Observe that they are all pinched at the origin, and that the lobe area in the first and third quadrants shrinks continuously to a straight line as ω increases, both being the fingerprint of memristors.

We conclude this tutorial by stressing that memristors are not inventions. They are discoveries and are ubiquitous. Indeed, many devices, including the "electric arc" dating back to 1801, have now been identified as memristors [19], [38]. Aside from serving as nonvolatile memories [20], *locally passive* memristors have been used for switching electromagnetic devices [21], for field-programmable logic arrays [22]–[26], for synaptic memories [27]–[29], for learning [30]–[32], etc.

In addition, *locally passive* memristors have been found to exhibit many exotic dynamical phenomena, such as oscillations [33], chaos [34], [35], Hamiltonian vortices [36], autowaves [37], etc.

V. CONCLUDING REMARKS

Any two-terminal device which exhibits a pinched hysteresis loop in the v-i plane when driven by any bipolar periodic voltage or current waveform, for *any* initial conditions, is a *memristor*. The loop shrinks to a straight line whose slope depends on the excitation waveform, as the excitation frequency tends to infinity.

Except in ideal cases, memristors, memcapacitors, and meminductors do *not* behave like resistors, capacitors, and inductors, respectively. For example, the potassium and sodium ion channel memristors in the Hodgkin–Huxley axon circuit model behave like *R–L* circuits [39]. It is conceptually wrong and misleading to identify memristors, memcapacitors, and meminductors with resistors, capacitors, and inductors. Each (α, β) element is a distinct circuit element because it cannot be built from the other elements.

Readers who may have been misled by some erroneous commentary in the popular press which associates an earlier gadget called a memistor with the memristor are referred to a technical clarification in [40].

We end this tutorial with the following succint signature of a memristor [14]:

If it's pinched it's a memristor. ■

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